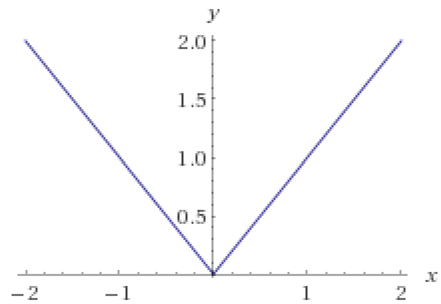
Review: Average Rate of Change - 9/16/16

1 Even and Odd Functions

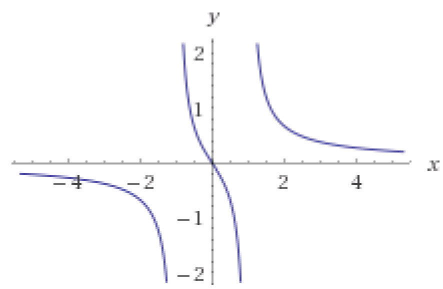
Definition 1.0.1 *A function f is **even** if $f(-x) = f(x)$. It is symmetric across the y axis.*

Example 1.0.2 *Let $f(x) = |x|$. $f(-x) = |-x| = |x| = f(x)$, so f is even.*



Definition 1.0.3 *A function f is **odd** if $f(-x) = -f(x)$. It is symmetric about the origin.*

Example 1.0.4 *Let $f(x) = \frac{x}{x^2-1}$. $f(-x) = \frac{-x}{(-x)^2-1} = -\frac{x}{x^2-1} = -f(x)$, so f is odd.*



Practice Problems
Are the following functions even, odd, or neither?

1. $f(x) = x^4 + x^2 - 2$.
2. $g(x) = x^3 - x$.
3. $h(x) = x^5 - 5$.

2 Average Rate of Change

Definition 2.0.5 *A* ***secant line*** *is a line that connects two points on a curve. Calculating the slope of the secant line gives us the* ***average rate of change.***

If we are looking at a graph of distance vs. time, then the average rate of change can be interpreted as the average velocity:

$$\text{average velocity} = \frac{\text{change in position}}{\text{change in time}}.$$

Example 2.0.6 *The Dartmouth Coach runs from Hanover to Boston Logan airport, and it stops on the way in Lebanon, New London, and South Station. Below is a time table of the Dartmouth Coach schedule.*

Location	***Time Elapsed***	***Distance Traveled***
Hanover	*0 hours*	*0 miles*
Lebanon	*1/3 hours*	*5 miles*
New London	*5/6 hours*	*30 miles*
South Station	*17/6 hours*	*130 miles*
Logan Airport	*3 hours*	*134 miles*

What is the average velocity of the coach between New London and South Station?

$$\begin{aligned} average\ velocity &= \frac{130 - 30 miles}{\frac{17}{6} - \frac{5}{6} hours} \\ &= \frac{100 miles}{2 hours} \\ &= 50 mph \end{aligned}$$

Practice Problems

A rock is thrown on Mars. It's height in feet after t seconds is given by $y = 10t - 1.86t^2$. Find the average velocity over each of the time intervals:

1. $[1, 2]$
2. $[1, 1.5]$
3. $[1, 1.1]$

3 Library of Functions

3.1 Constant and Linear Functions

Definition 3.1.1 *A* ***constant function*** *is of the form $f(x) = a$ for some constant a.*

Example 3.1.2 $f(x) = 5$ *is a constant function.*

Definition 3.1.3 *A* ***linear function*** *is a line. It is of the form $f(x) = mx + b$ where m is the slope and b is the y intercept.*

Example 3.1.4 $f(x) = -x + 3$ *is a linear function.*

3.2 Power Functions

Definition 3.2.1 *A **power function** looks like $f(x) = x^a$ for some constant a.*

Example 3.2.2 *$f(x) = x^3$ is a power function.*
$g(x) = x^{-3/4}$ is a power function.
$h(x) = x^2 + 7$ is NOT a power function.

3.3 Polynomial Functions

Definition 3.3.1 *A **polynomial function** looks like $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ where n is a nonnegative integer and a_i are real numbers. If $n \neq 0$, then the **degree** of the P is n.*

Example 3.3.2 *$f(x) = x^2 + 4x - 3$ is a polynomial function of degree 2.*
$g(x) = \pi x^3 + \sqrt{7}$ is a polynomial function of degree 3. Here $a_3 = \pi$, $a_2 = a_1 = 0$, and $a_0 = \sqrt{7}$.
$h(x) = x^{3/2}$ is NOT a polynomial function because $3/2$ is not an integer.
$k(x) = x^{-2}$ is NOT a polynomial function because -2 is negative.

3.4 Rational Functions

Definition 3.4.1 *A **rational function** is the quotient of two polynomials:*

$$R(x) = \frac{P(x)}{Q(x)}$$

where P and Q are both polynomials.

Example 3.4.2 *$f(x) = \frac{x^2+3}{x^3+x}$ is a rational function.*
$g(x) = \frac{x-5}{\sqrt{x}+7}$ is NOT a rational function because the denominator is not a polynomial.

3.5 Algebraic Function

Definition 3.5.1 *An **algebraic function** is a sequence of operations (addition, subtraction, multiplication, division, or taking roots) performed on a polynomial.*

Example 3.5.2 *$f(x) = \sqrt{x^2 - 5} \cdot \frac{x^3+7}{x^5 - \sqrt{x - \pi}}$ is an algebraic function.*
$g(x) = \sqrt[4]{x^2 - 25}$ is an algebraic function.
$h(x) = \sin(x)$ is NOT an algebraic function.

3.6 Floor and Ceiling Functions

Definition 3.6.1 *A **floor function** rounds the value of the function down to the nearest integer. The notation for a floor function looks like this: $f(x) = \lfloor x \rfloor$. A **ceiling function** rounds the value of the function up to the nearest integer. The notation for a ceiling function looks like this: $f(x) = \lceil x \rceil$.*